Discrete light localization in one-dimensional nonlinear lattices with arbitrary nonlocality

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(Received 21 March 2005; revised manuscript received 27 September 2005; published 16 December 2005)

We model discrete spatial solitons in a periodic nonlinear medium encompassing any degree of transverse nonlocality. Making a convenient reference to a widely used material—nematic liquid crystals—we derive a form of the discrete nonlinear Schrödinger equation and find a family of discrete solitons. Such self-localized solutions in optical lattices can exist with an arbitrary degree of imprinted chirp and have breathing character. We verify numerically that both local and nonlocal discrete light propagation and solitons can be observed in liquid crystalline arrays.

DOI: 10.1103/PhysRevE.72.066608 PACS number(s): 42.65.Tg, 42.65.Jx, 42.70.Df

I. INTRODUCTION

Optical energy localization in lattices has become an important branch of contemporary nonlinear science, due to a wealth of basic physics and potentials for light switching and logics [1–3] (and references therein). Attention has been recently devoted to light propagation in the frame of *tunable discreteness*, i.e., in lattices with an adjustable period, index contrast, nonlinearity. Examples to this extent are waveguide arrays in photorefractives [1], and droplet arrays of a Bose Einstein condensate [4–7]. In nematic liquid crystals (NLC), materials encompassing a large nonresonant reorientational response, spectrally extended transparency, strong birefringence, and mature technology [8], excitation as well as switching/steering of discrete solitons has been reported in voltage-tunable geometries [9–11]. Discrete solitons in NLC result from the interplay between evanescent coupling (owing to discreteness), molecular nonlinearity (leading to progressive mismatch as the extraordinary index increases), and nonlocality (owing to intermolecular elastic forces). The latter aspect, despite its role in several systems [12–22] including NLC [23,24], has only been discussed in the framework of one-dimensional (1D) discrete lattices, with reference to first order contributions [20] and long range dispersive interactions [19,21,22]. A general description of discrete solitons in the presence of a transverse nonlinear nonlocality is still lacking.

In this paper, with explicit reference to a physical system of interest—i.e., nematic liquid crystals—we model discrete light localization in media with an arbitrary degree of nonlocality, elucidating the interplay between nonlocality and nonlinearity on soliton dynamics. Using coupled mode theory (CMT) to derive the governing equations [25], we demonstrate the existence of a family of chirp-imprinted discrete breathers which could not be sustained by a purely local response. Finally, we verify the theoretical predictions by numerical experiments with a standard NLC. The paper is organized as follows. Section II introduces a model of liquid crystals and carries out an original reduction to a nonintegrable discrete nonlinear nonlocal Schrödinger equation, outlining the novelties with respect to previous studies dealing with nonlocality. In Sec. III we apply a variational approach using a convenient soliton ansatz and derive the differential equations for the evolution of soliton parameters in propagation. We highlight the impact of nonlocality on soliton generation and introduce a family of discrete chirped solitary waves. Finally, in Sec. IV, we perform a full numerical simulation of the actual liquid crystalline system and demonstrate the excellent agreement with our analytical predictions. We conclude by emphasizing how the examined NLC lattice offers the rare possibility to observe both local and nonlocal light propagation in one and the same system.

II. THEORETICAL APPROACH

We consider light propagation in a thin film planar waveguide of nematic liquid crystals, subject to a periodic transverse modulation along y and across x (Fig. 1). NLC consist of rodlike molecules which, electrically polarized across x, react to and reorient toward the field vector in order to minimize the free energy [8]. Under "planar" anchoring conditions at top and bottom interfaces of the cell, the mean angular orientation of the NLC molecules (i.e., the molecular director) is conveniently described by their angle θ with the axis z in the plane (x,z), as sketched in Fig. 1. This identifies the NLC optic axis with respect to the propagation wave vector of a light beam injected in the cell. If $n_a^2=n_\parallel^2-n_\perp^2$ is the NLC optical birefringence (with $n_\parallel$ and $n_\perp$ along or orthogonal to the *director*, respectively), an electric field (static or low frequency) applied across x, constant in z and periodic along y (Fig. 1), can reorient the director and determine a one-dimensional optical lattice with index modulation $n^2(x,y)=n_\perp^2+n_a^2\sin^2\theta(x,y)$ for e-polarized light. We assume an applied electric field $E_x(x,y)=E_0\sqrt{1+\epsilon F(y)}$, with a zero mean-value $F(y)=F(y+\Lambda)$ and an arbitrary $\epsilon<1$. In actual experiments, E_x is determined by the bias $V(x,y)\propto x[1+V(y)]$, with $V(y)=V(y+\Lambda)$ applied through an array of parallel finger-electrodes [9].

In the framework of the elastic continuum theory [8], the director distribution $\theta_0(x,y)$ at rest—i.e., with no injected

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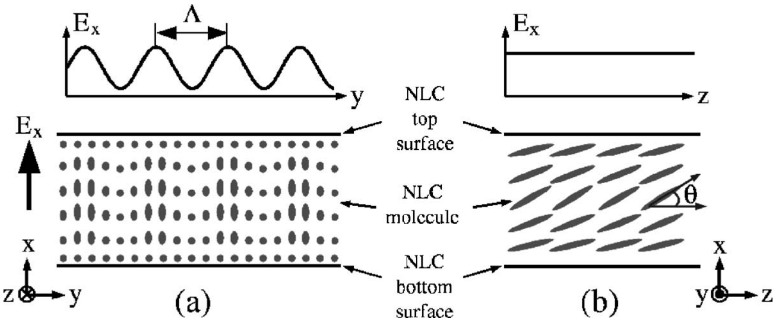

FIG. 1. Example of a discrete optical lattice in a cell of thickness d filled with undoped nematic liquid crystals: (a) front view; (b) side view. A periodically varying electric field E_x is applied through an electrode array across x and alters the mean molecular angular orientation $\theta(x,y)$, inducing an index modulation with the same period Λ. The top graphs sketch the distribution E_x versus y (left) and versus propagation z (right), respectively.

light—can be obtained by minimizing the NLC energy functional:

$$K\nabla_{xy}^2\theta_0 + \frac{\Delta\epsilon_{RF}|E_x|^2}{2}\sin(2\theta_0) = 0, \tag{1}$$

K being the NLC elastic constant (single constant approximation [8]) and $\Delta\epsilon_{RF}$ the (low frequency) anisotropy. When an e-polarized optical beam of slowly varying envelope A propagates in the medium, Eq. (1) modifies into

$$K\nabla_{xy}^2\theta + \left(\frac{\Delta\epsilon_{RF}|E_x|^2}{2} + \frac{n_a^2\epsilon_0|A|^2}{4}\right)\sin(2\theta) = 0. \tag{2}$$

The overall director distribution can be written as $\theta(x,y) = \theta_0(x,y)[1+\psi(x,y)]$, with $\theta_0(x,y)\psi(x,y)$ the nonlinear optical contribution. In usual experiments θ_0 is small ($\leqslant 0.4$), hence first order approximations are justified [9]. The nonlocality [linked to the ∇ operator in Eqs. (1) and (2)] has a different impact along x and y, respectively, due to the strong asymmetry of the problem (Fig. 1). A bell-shaped beam with an x waist comparable with the cell thickness d does not experience a nonlocal response along x, owing to the planar anchoring with $\theta_0 \approx 0$ in $x = \pm d/2$. Conversely, along y no anchoring is present and the index perturbation is free to widen. After substituting θ into Eqs. (1) and (2), assuming the response to be weakly nonlinear ($\psi \ll 1$) and local in x [$\partial_x^2(\theta_0\psi) \approx 0$], we obtain

$$\frac{\partial^2\psi}{\partial y^2} + \frac{2}{\theta_0}\frac{\partial\theta_0}{\partial y}\frac{\partial\psi}{\partial y} - \frac{4\Delta\epsilon_{RF}|E_x|^2\theta_0^2}{3K}\psi + \frac{n_a^2\epsilon_0|A|^2}{2K} = 0. \tag{3}$$

Equation (3) models the all-optical response of the NLC lattice. It can be cast in the integral form $\psi = \iint G(\zeta - x, \eta - y)|A(\zeta,\eta)|^2 d\zeta d\eta$, with $G(x,y)$ the Green's function. For a sufficient bias to induce an array of single-mode channel waveguides with nearest-neighbor coupling, CMT (in the tight-binding approximation) yields the z evolution of each eigenmode

$$i\frac{\partial Q_n}{\partial z} + C(Q_{n+1} + Q_{n-1}) + Q_n\sum_m \Gamma_{m,n}|Q_m|^2 = 0, \tag{4}$$

with $A = \Sigma_n Q_n(z) f_Q(x,y)\exp(-i\beta z)$, f_Q and β the modal eigenfunction and eigenvalue, respectively, $|Q_n|^2$ the mode power in the nth channel, C the coupling strength, and

$$\Gamma_{m,n} = \frac{\omega\epsilon_0}{4}\int |f_Q(x,y-n\Lambda)|^2 n_{2,\psi}(x,y) G(\zeta - x, \eta - y) \times |f_Q(\zeta, \eta - m\Lambda)|^2 dxdyd\zeta d\eta \tag{5}$$

the nonlinear overlap integral, with $n_{2,\psi}(x,y) = n_a^2\sin(2\theta_0)\theta_0/2n(\theta_0)$. Factorizing the Green's function as $G(x,y) = G(x)G_p(y)G_e(y)$, with $G_p(y) = G_p(y+\Lambda)$ and the envelope $G_e(y)$ wider than the guided mode, the integral (5) becomes $\Gamma_{m+n} = G_e[(m+n)\Lambda]\Upsilon$, with

$$\Upsilon = \frac{\omega\epsilon_0}{4}\int |f_Q(x,y)|^2 n_{2,\psi}(x,y) G(\zeta - x) G_p(\eta - y) \times |f_Q(\zeta,\eta)|^2 dxdyd\zeta d\eta. \tag{6}$$

To proceed with the analysis, we need to calculate the Green's function from Eqs. (1) and (3). By setting $(x,y) = (Xd, Y\Lambda)$, $\theta_0 = \theta_r\vartheta(X,Y)$, $\vartheta(0,0) = 1$, $\psi = 1/(k_0^2 n_a^2\Lambda^2) \times \phi(X,Y)$, $A^2 = 8/\sqrt{3}(\sqrt{\Delta\epsilon_{RF}K}E_0/\epsilon_0 k_0^2 n_a^4\theta_r^2\Lambda^3)a^2(X,Y)$, $\phi = \iint g(\zeta - X, \eta - Y)/\vartheta(\zeta - X, \eta - Y)|a(\zeta,\eta)|^2 d\zeta d\eta$ [scaling g to ϑ prevents first order derivatives to appear in Eq. (8)], $F = (4\theta_r^2/3\alpha)f(Y)$, $\alpha = \Lambda^2/R_c^2$, $R_c^2 = 3K/4\Delta\epsilon_{RF}E_0^2\theta_r^2$, $\beta = \Lambda^2/d^2$, and $\gamma(\epsilon) = [\alpha + \epsilon(4\theta_r^2 f/3)]\vartheta^2 + \partial^2\vartheta/\partial Y^2\vartheta$, Eq. (3) with Eq. (1) can be cast in the dimensionless form

$$\frac{\partial^2\vartheta}{\partial Y^2} + \beta\frac{\partial^2\vartheta}{\partial X^2} + \left(\frac{3}{4\theta_r^2}\alpha + \epsilon f\right)\vartheta = 0, \tag{7}$$

$$\frac{\partial^2 g}{\partial Y^2} - \gamma(\epsilon)g + 2\sqrt{\alpha}u_0 = 0. \tag{8}$$

In order to solve Eq. (7) above, we separate the variables X and Y by letting $3\alpha/4\theta_r^2 = \upsilon_x + \upsilon_y$ and obtain a simple form of Hill's equation in Y

$$\frac{d^2\vartheta}{dY^2} + (\upsilon_y + \epsilon f)\vartheta = 0 \tag{9}$$

with $\vartheta(X,Y) = \vartheta(X)\vartheta(Y)$ and $\vartheta(X) \propto \sin(\pi X + \pi/2)$ corresponding to a harmonic oscillator across X. According to Floquet theory, the periodic solutions $\vartheta(Y) = \vartheta(Y+1)$ are located on a transition curve [26]. We adopt the perturbative method of strained parameters [26,27], performing the expansion:

$$\vartheta = \sum_m \epsilon^m \vartheta_m(Y), \tag{10}$$

$$v_y = \sum_m \epsilon^m v_{y0},$$

with $\vartheta_m(Y)=\vartheta_m(Y+1)$. By collecting and equating terms of the same order in ϵ, we find

$$\frac{\partial \vartheta_0(Y)^2}{\partial Y^2} = 0 \quad O(1), \tag{11}$$

$$\frac{\partial \vartheta_1(Y)^2}{\partial Y^2} = - v_{y1}\vartheta_0 - f\vartheta_0 \quad O(\epsilon), \tag{12}$$

$$\frac{\partial \vartheta_2(Y)^2}{\partial Y^2} = - v_{y2}\vartheta_0 - v_{y1}\vartheta_1 - f\vartheta_1 \quad O(\epsilon^2). \tag{13}$$

By expanding $f(Y)$ in a Fourier basis $f=\Sigma_m \chi_m \exp(-i2\pi mY)$, we obtain the solution to Eqs. (11)–(13)

$$\vartheta = 1 + \sum_m \epsilon^m \vartheta_m(Y) = 1 + \epsilon \sum_m \frac{\chi_m}{4\pi^2 m^2}\exp(- i2\pi mY) + O(\epsilon^2), \tag{14}$$

$$v_y = - \epsilon^2 \int \vartheta_1(Y)f(Y)dY + O(\epsilon^3). \tag{15}$$

Substituting Eq. (14) into Eq. (8), for $\vartheta(X)\approx$constant within the effective modal width, we get

$$\frac{\partial^2 g}{\partial Y^2} - [\alpha + \sum_m \epsilon^m h_m(Y)]g + 2\sqrt{\alpha}u_0 = 0 \tag{16}$$

with $g(X,Y)=u_0(X)g(Y)$, $h_1=f(Y)[(4\theta_r^2/3)-(1/\vartheta(Y))]+2\alpha\vartheta_1(Y)$ and a periodic $h_m(Y)=h_m(Y+1)$ the expression of which stems from Eqs. (11)–(14). For $u_0=0$, Eq. (16) is Hill's equation; otherwise for $u_0\neq 0$ its solutions can be found through the expansion

$$g(Y) = \sum_m \epsilon^m g_m(Y). \tag{17}$$

Substituting and equating terms of the same order in ϵ, we have

$$\frac{\partial g_0^2}{\partial Y^2} - \alpha g_0 + 2\sqrt{\alpha}u_0 = 0 \quad O(1), \tag{18}$$

$$\frac{\partial g_1^2}{\partial Y^2} - \alpha g_1 = h_1 g_0 \quad O(\epsilon), \tag{19}$$

$$\frac{\partial g_2^2}{\partial Y^2} - \alpha g_2 = h_1 g_1 + h_2 g_0 \quad O(\epsilon^2). \tag{20}$$

At each order the solution can be factorized in the form $g(Y)=g_e(Y)g_{pn}(Y)$, i.e., an envelope $g_e(Y)$ modulated by the periodic function $g_{pn}(Y)=g_{pn}(Y+1)$, with

$$g_e(Y) = \exp(- \sqrt{\alpha}|Y|),$$

$$g_{p0} = 1,$$

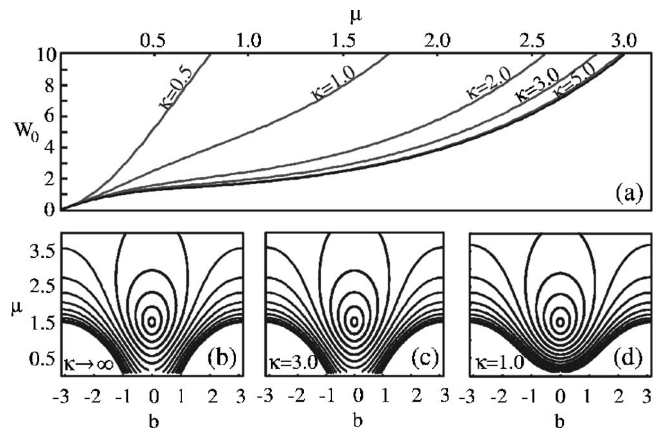

FIG. 2. (a) Power W_0 of discrete soliton versus size μ for increasing κ (decreasing nonlocality) up to the local Kerr case (thick line), (b)–(d) phase planes of soliton chirp b and width μ for various κ in the case $\mu=1.5$.

$$g_{p1}(Y) = - \sum_m \frac{h_{1m}}{4m(\alpha + m^2\pi^2)}\Bigg[m + 3m \exp(- i2\pi mY) + \frac{2\,\mathrm{sign(Y)}}{\pi\sqrt{\alpha}}\exp(- i\pi mY)\sin(m\pi Y)\Bigg]. \tag{21}$$

Therefore, the generic solution $g(Y)$ can be written in the form $g_n(Y)=g_e(Y)g_p(Y)$, with a peaked envelope $g_e(Y)$ and a periodic modulation $g_p(Y)=\Sigma_n \epsilon^n g_{pn}$. Now, after introducing the (dimensionless) fields $q_n=Q_n\sqrt{\Upsilon \coth(\kappa/2)/2C}\times\exp(-i2\xi)$ with $\xi=Cz$ and the degree of (non)locality $\kappa=\sqrt{\alpha}$, by substituting Eqs. (21) in Eq. (4) we finally obtain the discrete nonlocal nonlinear Schrödinger equation

$$i\frac{\partial q_n}{\partial \xi} + (q_{n+1} + q_{n-1} - 2q_n) + 2q_n \sum_m \frac{\exp(- \kappa|m + n|)}{\coth\kappa/2}|q_m|^2 = 0. \tag{22}$$

Equation (22) is a discrete version of the nonlocal nonlinear Schrödinger equation (NNLS) [18] and reduces to the discrete nonlinear Schrödinger equation (DNLS) [28] as $\kappa\to\infty$. At variance with previous work dealing with nonlocality in dispersion (a linear property of the medium) [19,21,22], Eqs. (22) address a *nonlinear* feature (nonlocality in the nonlinear response) and, thereby, are expected to possess a radically distinct dynamics with respect to their linear counterpart (see Sec. III) and to nonlocal models in the frame of nonlinear photonic crystals ([15] and references therein). The lack of propagation terms in Eqs. (8) and (9) of [15], for instance, does not allow to study the system evolution (as investigated hereby). Hence, Eqs. (22) can be regarded as a general model of discrete, dispersive, nonlinear nonlocal media.

III. ANALYSIS OF THE DISCRETE MODEL

Equation (22) can also be derived using the variational principle and the following Lagrangian L:

$$L = \sum_n \frac{i}{2}\left(q_n^* \frac{\partial q_n}{\partial \xi} - q_n \frac{\partial q_n^*}{\partial \xi}\right) - |q_{n+1} - q_n|^2 + \sum_m \frac{\exp(-\kappa|m+n|)}{\coth\frac{\kappa}{2}} |q_m|^2 |q_n|^2. \quad (23)$$

We adopt the peaked ansatz $q_n(\xi) = A_q(\xi)\exp[i\varphi(\xi) + ib(\xi)|n-n_0| - \mu(\xi)|n-n_0|]$ with $n_0=0$, obtaining the effective Lagrangian L_{eff}

$$L_{\text{eff}} = A_q^2\left[-\frac{\partial \varphi}{\partial \xi}\coth\mu - \frac{1}{2}\frac{\partial b}{\partial \xi}\frac{1}{\sinh\mu} - 2\coth\mu + 2\frac{\cos b}{\sinh\mu} + A_q^2 N(\mu,\kappa)\right] \quad (24)$$

with nonlocal contribution $N(\mu,\kappa)$

$$N(\mu,\kappa) = \frac{1}{A_q^4}\sum_{m,n}\frac{\exp(-\kappa|m+n|)}{\coth(\kappa/2)}|q_m|^2|q_n|^2 = \frac{\tanh(\kappa/2)[2\sinh 2\mu + \sinh\kappa + \sinh(4\mu+\kappa)]}{4\sinh 2\mu \sin^2(\mu+\kappa/2)} \quad (25)$$

as calculated with a bilateral Z–transform. Setting to zero each variational derivative of soliton parameters (A_q, φ, b, μ), we obtain the evolution of both soliton chirp b and soliton width μ. After some algebra

$$\frac{\partial b}{\partial \xi} = \frac{4\cos b \sinh^3\mu}{\cosh 2\mu} - W_0 \frac{\sinh^2\mu\left[4N(\mu,\kappa)\tanh\mu + 2\sinh^2\mu\frac{\partial N(\mu,\kappa)}{\partial \mu}\right]}{\cosh 2\mu}, \quad (26)$$

$$\frac{\partial \mu}{\partial \xi} = -\frac{4\cosh\mu \sin b \sinh^2\mu}{\cosh(2\mu)}, \quad (27)$$

with the soliton power $W_0 = \Sigma_n |q_n|^2 = A_q^2 \coth\mu$. Equations (26) and (27) define a two-dimensional phase space with the conserved Hamiltonian H_{eff}

$$H_{\text{eff}} = -W_0\left[-2 + 2\cos b \,\text{sech}\,\mu + W_0 \frac{\tanh(\kappa/2)\tanh^2\mu[2\sinh 2\mu + \sinh\kappa + \sinh(4\mu+\kappa)]}{4\sinh 2\mu \sin^2(\mu+\kappa/2)}\right]. \quad (28)$$

Solitons correspond to stationary points of Eqs. (26) and (27) with $b=0$ and

$$W_0 = \frac{8\cosh^2\mu\cosh\frac{\kappa}{2}}{3\sinh^2\frac{\kappa}{2}} \times \frac{\sinh\mu\sinh^3\left(\mu+\frac{\kappa}{2}\right)}{[\sinh\mu + \sinh(3\mu+\kappa)(-1+2\cosh 2\mu)]}. \quad (29)$$

Since $(\partial W_0(\mu,\kappa)/\partial\mu) > 0$, all fixed points representing solitons are stable [28]. As shown in Fig. 2(a), the existence curve (29) of discrete solitons rapidly approaches the local Kerr case for diminishing nonlocality (i.e., increasing $\kappa > 3$). As nonlocality is enhanced and κ reduces toward and below 1, however, the refractive perturbation becomes broader and broader (in y) and W_0 larger and larger. Substantial changes are visible near the soliton solution, as displayed in the phase plane [29] of Eqs. (26) and (27) in Figs. 2(b)–2(d). In a Kerr regime ($\kappa \to \infty$) the phase plane consists of a series of periodic orbits near the localized state ($\mu=1.5$, $b=0$) and μ tends to zero for higher chirps b [Fig. 2(b)]. Therefore, the addition of an initial chirp above a certain value—i.e., enough *chirp imprinting*—destroys the soliton [5,30,31]. The situation keeps unchanged as $\kappa \geqslant 3$ [Fig. 2(c)]. In the nonlocal regime ($\kappa=1.0$), conversely, the trajectories evolve from a closed loop to a limit cycle, hence no *chirp imprinting* can break the soliton [Fig. 2(d)]. This remarkable finding is confirmed by numerical simulations, as visible in Fig. 3. While a local system cannot sustain discrete light localization with an input spatial chirp above a threshold [Fig. 3(a)], nonlocality allows for the propagation of chirped discrete solitary waves

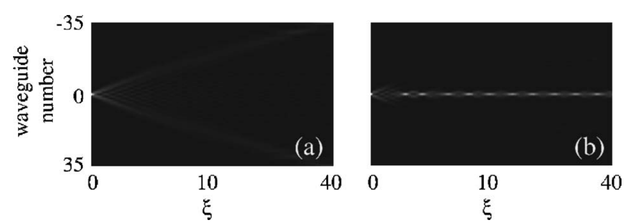

FIG. 3. (a) Discrete soliton breaking in a quasilocal regime ($\kappa=3.0$) and (b) breatherlike propagation in the nonlocal regime ($\kappa=1.0$) for $\mu=1.5$ and an imprinted chirp $b=3$.

[Fig. 3(b)] with periodically varying width, as predicted by our model. Clearly, the soliton amplitude oscillates as well in order to conserve the total power W_0. These solutions belong to the class of discrete breathers and are the lattice counterparts of the continuously breathing solitons reported in highly nonlocal bulk NLC [24].

IV. NUMERICAL EXPERIMENTS WITH THE NLC LATTICE

In order to link the analysis to an actual NLC lattice, we need to estimate the range of available κ. As it stems from the model above, the nonlinear index change has a peaked envelope of transverse size $\kappa = \Lambda / R_c \propto \Lambda V / \sqrt{Kd}$ [being $E_x(x) \approx V/d$ and $\theta_r \approx$ const] and, therefore, nonlocality can be tuned by acting on either one of the form-factor Λ / d, the bias V, the elastic constant K, or the temperature [32]. With reference to a standard NLC (nematic 5CB, with $K = 3.8 \times 10^{-12}$ N), in order to evaluate the Green's function along Y we employed an optical ($\lambda = 1.064$ μm) excitation $A(x,y)$ consisting of a Dirac distribution across y and a Gaussian beam of waist $\approx d$ across x. We numerically integrated Eqs. (1) and (2) using the relevant potential distribution (see Ref. [10]), and finally derived the size κ versus $\sigma \equiv \Lambda V / \sqrt{Kd}$ by fitting the calculated reorientation profile. As the material is tuned from local [Fig. 4(a) with $d/\Lambda = 0.44$, $V = 0.78$ V] to nonlocal [Fig. 4(b) with $d/\Lambda = 1.0$, $V = 0.75$ V], the Green's function envelope of the nonlinear reorientation along y (solid line) is well approximated by our theoretical model (dashed line). In the latter analysis we employ both geometric (d/Λ) and material (V) tuning, slightly adjusting the bias to keep $\theta_r \approx 0.35$. This results in a quasilinear transition [see Fig. 4(c)] from a local to a nonlocal response as σ varies. The corresponding transverse size of the nonlocal response, represented by dots in Fig. 4(c), shows that theory and numerics are in excellent agreement in the range covering local ($\kappa = 3.0$) to nonlocal ($\kappa = 1.0$) responses. Moreover, the all-optical reorientation across x, visible in Fig. 4(d), is nearly sinusoidal (dashed line) and does not widen significantly when the nonlinearity intervenes (solid line), supporting the validity of the local approximation previously adopted.

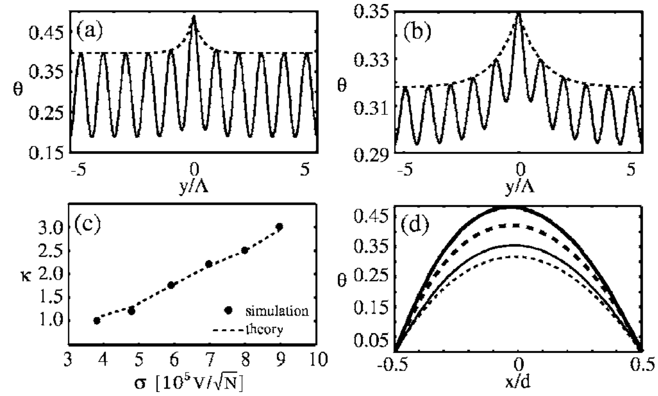

FIG. 4. Calculated reorientation distribution along y ($x = 0$) (solid line) and theoretical Green's function envelope (dashed line) in (a) the local ($\sigma = 3.85 \times 10^5$ V/$\sqrt{\text{N}}$) and in (b) the nonlocal ($\sigma = 9 \times 10^5$ V/$\sqrt{\text{N}}$) cases, respectively; (c) corresponding degree of nonlocality κ versus σ after a fit (dots) and from theory (dashed line); (d) reorientation profile versus x ($y = 0$) without (dashed lines) and with (solid lines) optical excitation (1 mW) for local (thick lines) and nonlocal responses (thin lines), respectively.

V. CONCLUSIONS

In conclusion, with specific reference to nematic liquid crystals and their reorientational response, we have modeled discrete light localization in a nonlinear medium with an arbitrary degree of transverse nonlocality. Starting from the governing equation of the liquid crystalline system, we performed an original reduction to a general form of discrete nonlinear nonlocal Schrödinger equation. Remarkably, the latter result was not achieved by introducing an *a priori* specific from of nonlocality [18–22], but one derived from the molecular response of NLC. We employed a variational procedure and investigated the role of nonlocality in supporting chirp-imprinted discrete spatial solitons. Such solutions are periodic breathers and cannot exist in purely local systems. Since the degree of nonlocality in NLC arrays can be adjusted by acting on geometric or material or external parameters [32], we anticipate that our findings will trigger the observation of discrete light propagation in both local and nonlocal regimes in one and the same system. Our numerical experiments, in excellent agreement with the theoretical predictions, fully support such a possibility.

ACKNOWLEDGMENTS

We acknowledge enlightening discussions with C. Conti and D. Levi.

[1] D. N. Christodoulides, F. Lederer, and Y. Silberberg, Nature (London) **424**, 817 (2003).
[2] P. G. Kevrekidis, K. Rasmussen, and A. R. Bishop, Int. J. Mod. Phys. B **15**, 2833 (2001).
[3] J. W. Fleischer, M. Segev, N. K. Efremidis, and D. N. Christodoulides, Nature (London) **422**, 147 (2003).
[4] A. Trombettoni and A. Smerzi, Phys. Rev. Lett. **86**, 2353 (2001).
[5] F. K. Abdullaev, E. N. Tsoy, B. A. Malomed, and R. A. Kraenkel, Phys. Rev. A **68**, 053606 (2003).
[6] A. Anderson and M. A. Kasevich, Science **282**, 1686 (2001).
[7] Y. V. Kartashov, L. C. Crasovan, A. S. Zelenina, V. A. Vysloukh, A. Sanpera, M. Lewenstein, and L. Torner, Phys. Rev. Lett. **93**, 143902 (2004).
[8] I. C. Khoo, *Liquid Crystals: Physical Properties and Nonlinear Optical Phenomena* (Wiley, New York, 1995).
[9] A. Fratalocchi, G. Assanto, K. A. Brzdakiewicz, and M. A. Karpierz, Opt. Lett. **29**, 1530 (2004).

[10] A. Fratalocchi, G. Assanto, K. A. Brzdakiewicz, and M. A. Karpierz, Appl. Phys. Lett. **86**, 051112 (2005).
[11] A. Fratalocchi, G. Assanto, K. A. Brzdakiewicz, and M. A. Karpierz, Opt. Lett. **30**, 174 (2005).
[12] A. G. Litvak and A. M. Sergeev, JETP Lett. **27**, 517 (1978).
[13] H. L. Pecseli and J. J. Rasmussen, Plasma Phys. **22**, 421 (1980).
[14] S. K. Turitsyn, Theor. Math. Phys. **64**, 797 (1985).
[15] S. F. Mingaleev, Y. S. Kivshar, and R. A. Sammut, Phys. Rev. E **62**, 5777 (2000).
[16] V. M. Perez-Garca, V. V. Konotop, and J. J. Garcia-Ripoll, Phys. Rev. E **62**, 4300 (2000).
[17] A. V. Mamaev, A. A. Zozulya, V. K. Mezentsev, D. Z. Anderson, and M. Saffman, Phys. Rev. A **56**, R1110 (1997).
[18] O. Bang, W. Krolikowski, J. Wyller, and J. J. Rasmussen, Phys. Rev. E **66**, 046619 (2002).
[19] Y. Gaididei, N. Flytzanis, A. Neuper, and F. G. Mertens, Phys. Rev. Lett. **75**, 2240 (1995).
[20] Y. V. Kartashov, V. A. Vysloukh, and L. Torner, Phys. Rev. Lett. **93**, 153903 (2004).
[21] Y. B. Gaididei, S. F. Mingaleev, P. L. Christiansen, and K. O. Rasmussen, Phys. Rev. E **55**, 6141 (1997).
[22] H. Feddersen, Phys. Lett. A **154**, 391 (1991).
[23] C. Conti, M. Peccianti, and G. Assanto, Phys. Rev. Lett. **91**, 073901 (2003).
[24] C. Conti, M. Peccianti, and G. Assanto, Phys. Rev. Lett. **92**, 113902 (2004).
[25] D. N. Christodoulides and R. I. Joseph, Opt. Lett. **13**, 794 (1988).
[26] A. H. Nayfeh, *Introduction to Perturbation Techniques* (Wiley, New York, 1993).
[27] A. H. Nayfeh and D. T. Mook, *Nonlinear Oscillations* (Wiley, New York, 1995).
[28] B. Malomed and M. I. Weinstein, Phys. Lett. A **220**, 91 (1996).
[29] E. Ott, *Chaos in Dynamical Systems* (Cambridge University Press, Cambridge, 1997).
[30] D. J. Kaup, J. El.-Reedy, and B. Malomed, Phys. Rev. E **50**, 1635 (1994).
[31] M. Desaix, L. Helczynski, D. Anderson, and M. Lisak, Phys. Rev. E **65**, 056602 (2002).
[32] M. Peccianti, C. Conti, and G. Assanto, Opt. Lett. **30**, 415 (2005).